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LETTER TO THE EDITOR

The three-dimensional *XY* model in *P*-fold random anisotropy

P Reed

School of Computer Studies and Mathematics, Sunderland Polytechnic, Sunderland SR1 3SD, UK

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Abstract. The three-dimensional *XY* model in the presence of two- and threefold random anisotropy has been investigated by Monte Carlo simulation and finite-size scaling using system sizes 4^3, 8^3, 16^3, 32^3. The results are consistent with the existence of a conventional transition to a spin glass phase at $T_c = 2.2$ and with correlation exponent $\nu = 0.67$. No evidence for quasiferromagnetism is found.

Randomness can have a profound effect on the critical properties of model magnetic systems. New phases can be produced and critical dimensionalities changed. A classic example of this is the much studied Edwards–Anderson model of a spin glass [1]. Here the randomness arises from fluctuations in both the magnitude and sign of the spin-coupling constant. Such randomness destroys ferromagnetism, but there is still a phase transition for the three-dimensional Ising spin glass and, for greater than three dimensions, for the equivalent Heisenberg system. However, the result for Heisenberg systems is still controversial [2].

Here the concern will be with systems where the randomness arises not from bond disorder but from random anisotropy fields acting on each spin. That is, each spin resides in an anisotropy field of the general form

$$h_1^i S^i + h_2^{ij} S^i S^j + h_3^{ijk} S^i S^j S^k + \ldots$$

where s^i is the spin component and the $\{h_u\}$ are a set of random u-rank tensors. Such terms arise from the interaction of the spin with the host, for example intermetallic compounds of rare earths and non-magnetic metals [3].

Random anisotropy models seem to have been studied much less than spin glasses and much of the work has been restricted to the case of second rank anisotropy which has certain simplifying features in the limit of an infinite anisotropy field. The critical properties induced by random anisotropy are not clear and several scenarios exist. What does seem to be agreed upon is that the domain-wall arguments of Imry and Ma [4] can be extended to the case of random anisotropy and that ferromagnetism is destroyed in less than four dimensions for continuous spin models [5].

Dotsenko and Feigelman [6] investigated the *XY* model in twofold anisotropy and have inferred the existence of spin glass type order and quasiferromagnetism†. However, Fisher [7], in an illuminating exposition, cast doubt on this finding since their perturbative method was being used in the non-perturbative region. Pelcovits *et*

† Quasiferromagnetism is the name given to a phase with zero ferromagnetic order parameter but which has algebraic decay of the correlation functions the same as in the pure two-dimensional *XY* model.

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al [8] have also predicted spin glass behaviour for twofold anisotropy, their results becoming exact in the limit of infinite spin dimensionality. However, it has been suggested [7] that the spin glass phase discussed in the literature is an artefact of the spherical limit.

Aharony and Pytte [9], investigating the twofold case, predict quasiferromagnetism, and this result has been extended to any anisotropy for the XY model [10]. More recently Villain and Fernandez [11] in a real space renormalization group study of the XY model excluding vortices also predicted quasiferromagnetism. However, Chudnovsky *et al* [5] investigated the ground state of the XY model in p-fold anisotropy and found no quasiferromagnetism in three dimensions.

Thus the situation is unclear. The problem has been investigated using Monte Carlo simulation and finite-size scaling for the XY model on a simple cubic lattice and in the presence of p-fold anisotropy. The Hamiltonian for such a system is

$$-J\sum_{ij}\cos(\theta_i-\theta_j)-D\sum_i\cos(p\theta_i+\phi_i)-1 \qquad (1)$$

where $0\leqslant\theta_i<2\pi$ are site variables and $\{\phi_i\}$ are quenched in random axes chosen uniformly between 0 and 2π. Some previous simulations of (1) have been restricted to the case of $D\to\infty$ and $p=2$ where the Hamiltonian becomes Ising-like [12, 13]. No such limit is taken here and the values $J=D=1$ are used.

The simulation has concerned itself with a search for spin glass order and or quasiferromagnetism, and estimating T_c and the exponent ν. Hamiltonian (1) has been simulated for $p=2$ and 3, and for the range of system sizes of linear dimension $N=4$, 8, 16, 32.

Spin glass order was investigated by calculating the overlap between spin configurations at time τ and t given by

$$q(t)=\frac{1}{N^3}\sum_i\cos(\theta_i(t)-\theta_i(\tau)) \qquad (2)$$

where τ and t are measures in steps per spin. The spin glass order parameter Q is related to (2) by

$$Q=\lim_{t\to\infty}q(t).$$

The calculation proceeded as follows. The system was first relaxed for a time τ and the spin configuration at that time stored. The system was then relaxed for a further time τ after which the probability distribution $P(q)$ was calculated by observing the distribution of $q(t)$ for 2τ further Monte Carlo steps. From the distribution of $P(q)$ the following scaling function was constructed

$$g=\tfrac{1}{2}(3-[q^4]/[q^2]^2). \qquad (3)$$

Where [ ] denote average over $P(q)$. Each simulation was initiated from a random spin configuration and the system slowly cooled in temperature steps of either $0.1J$ or $0.05J$. Results for g were averaged over between 12 and 25 distinct anisotropy field realizations for the larger sized systems and many more for the smaller systems. Underestimating the relaxation time τ would result in an overestimate of g. Thus to try to ensure that τ was sufficiently long the run time was increased and the effect on g monitored. In this way τ was estimated as the time required to produce stable g and beyond which increase in run time produced no discernible drift down in g. The stability of g was tested over tens of thousands of steps and in the case of the small systems by increasing the run time several fold. It was found that for the smaller systems the required run time was about 80 000 steps and for the larger systems 250 000 steps. Under these conditions the magnetization often reduced to a small residual with many changes of sign. This is taken to indicate that phase space is being well explored and that metastability is not the cause of the results reported. I would thus judge the time stability of the results reported here to be good.

The scaling function g has been successful in locating spin glass order [12, 14], and has the functional form

$$g = g(N^{1/\nu}(T - T_c)).$$

Plots of g against temperature are shown in figures 1 and 2 for the case of $p = 2$ and

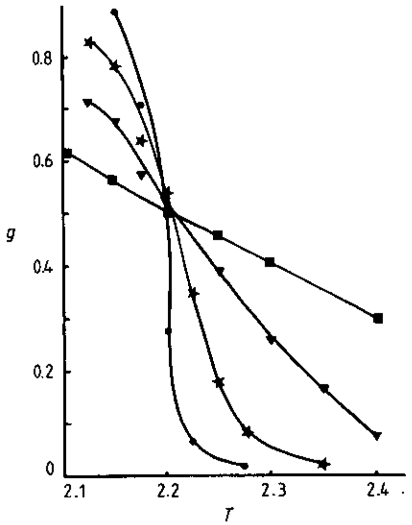

Figure 1. g (equation 3) against temperature T for the case of $p = 2$. The critical temperature is indicated by the crossing of the curves at approximately $T = 2.2$: squares $= 4^3$, triangles $= 8^3$, stars $= 16^3$, dots $= 32^3$. The lines are only visual guides.

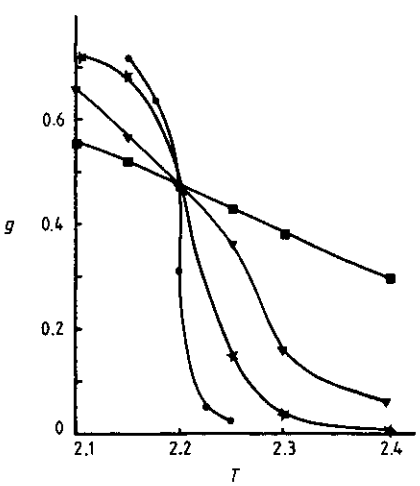

Figure 2. g (equation 3) against temperature T for the case $p = 3$. The critical temperature is indicated by the crossing of the curves at $T = 2.2$. The symbols have the same meaning as in figure 1 and the lines are only visual guides.

3 respectively. It is seen that at $T = 2.2$ the curves cross indicating that the results are size independent at this point and hence this temperature is the critical temperature T_c. It should also be noted that below T_c the curves for different N values fan out. This is to be contrasted with the $D \to \infty$ results found by Chakrobarti [12] where the curves coalesce for T less than T_c.

If the random anisotropy model is a quasiferromagnet then it might be expected that the temperature T_c would be the terminus of a line of critical temperatures as in the two-dimensional XY ferromagnet. In this case all temperatures below T_c are critical and the correlation length is always infinite. If such was the case then the plots for g would become size independent below T_c. This does not appear to be the case from the plots in figures 1 and 2, arguing against quasiferromagnetism. Also quasiferromagnets have infinite susceptibility at all temperatures below T_c, consequently the variable

$$S = \langle(\Sigma \cos \theta_i / N^3)^2\rangle \tag{4}$$

would behave as $N^{-(1+\eta)}$ below T_c. As a signature of quasiferromagnetism a search has been made for such behaviour. Figure 3 shows a log–log plot of S against N for T less than T_c; linearity would indicate quasiferromagnetism. The results shown in figure 3 do not support this conclusion as the plots are clearly nonlinear, again arguing against quasiferromagnetism.

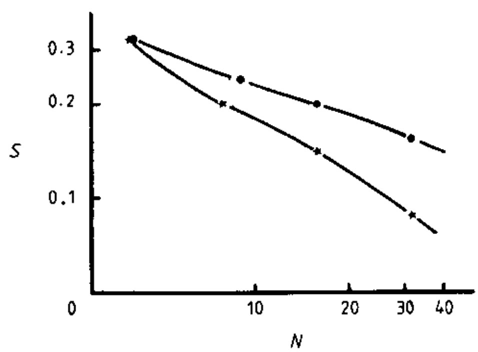

Figure 3. S (equation 4) against N, log–log scale. Dots indicate the plot for $p = 3$ and $T = 2.1$, stars indicate the plot for $p = 2$ and $T = 2.15$.

Figure 4 shows a plot of g against $N^{1/\nu}(T - T_c)$ for the values $T_c = 2.2$ and $\nu = 0.67$. The data points on this figure are the results of averaging over many hundreds of samples giving greater numerical stability but restricting the simulation to small system sizes. Inspection suggests that the plots lie along universal curves indicating that $T = 2.2$ and $\nu = 0.67$ are not inconsistent with scaling. These values for T_c and ν are of similar value to those found by high temperature series for the pure system [15].

In the two-dimensional XY model in random anisotropy Cardy and Ostlund [16] have predicted the existence of more than one phase transition for sufficiently large p. Simulation has given support to this conclusion [18]. It seems possible that in three dimensions there may be more than one phase transition for at least some p. It therefore could be that some of the predictions made for this model could still be true at lower temperatures. Because of extremely long relaxation times, low temperatures are difficult to access by Monte Carlo simulation.

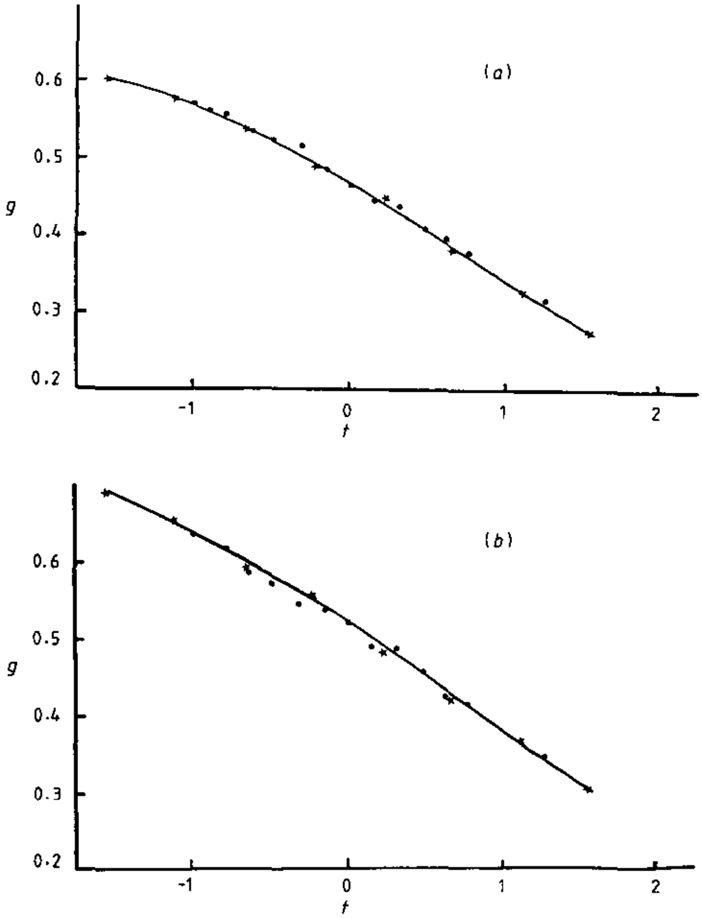

Figure 4. Scaling plots of g against $t = N^{1/\nu}(T - T_c)$ using $\nu = 0.67$ and $T_c = 2.2$: (a) is for $p = 3$, (b) is for $p = 2$. Dots $= 4^3$ and stars $= 8^3$.

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